CBCS Scheme

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16MCA15

First Semester MCA Degree Examination, Dec.2016/Jan.2017
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing one full question from each module.***

Module-1

1 a. Define (i) Tautology (ii) Contradiction. For any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to [p \to r]$ is a tautology. **(06 Marks)**

b. Establish the validity of the following argument:
$(q \vee \neg r) \vee s$
$\neg q \vee (r \wedge \neg q)$
$\therefore r \to s$ **(05 Marks)**

c. Give a direct proof of the statement "sum of two even integers is itself an even integer". **(05 Marks)**

OR

2 a. Without constructing the truth table, prove that $(p \to q) \to (\neg q \to \neg p)$. **(05 Marks)**

b. Prove that for any three propositions p, q, r, $p \to (q \vee r) \Leftrightarrow (p \to q) \vee (p \to r)$. **(05 Marks)**

c. Test the validity of the following argument:
$(\forall x)(B(x) \to \neg C(x))$; $(\exists x)(C(x) \wedge D(x))$; $(\exists x)(D(x) \wedge \neg B(x))$ **(06 Marks)**

Module-2

3 a. Define symmetric difference of the sets A and B. For any three sets A, B and C. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **(05 Marks)**

b. Define equivalence relation. Let a relation R be defined on the set Z by "aRb if a, b $\in Z$ and $a - b$ is divisible by 5". Is R an equivalence relation? Justify your answer. **(06 Marks)**

c. A computer company requires 30 programmers to handle system programming jobs and 40 programmers application programming. If the company appoints 55 programmers to carryout these jobs how many of these perform programming jobs? How many handle only application programming? **(05 Marks)**

OR

4 a. Prove that a function $f : A \to B$ is invertible if and only if it is one to one and onto. **(05 Marks)**

b. Let $f : R \to R$ and $g : R \to R$ where $f(x) = ax + b$, $g(x) = 1 - x + x^2$ if $(gof)(x) = 9x^2 - 9x + 3$ determine a and b. **(06 Marks)**

c. For any two sets A and B prove the De-Morgan's laws.
(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ **(05 Marks)**

Module-3

5 a. Find the number of permutations of the word "EXCELLENCE". How many of these permutations:
(i) Begin with E (ii) Begin with E and with C? **(06 Marks)**

b. Find the least number of ways of choosing three different numbers from 1 to 10 so that all choices have the same sum. **(05 Marks)**

c. Find the homogeneous solution of the recurrence relation,
$a_r - 5a_{r-1} + 6a_{r-2} = 2r$. **(05 Marks)**

OR

6 a. A group consists of 4 girls and 7 boys. In how many ways a team of 5 members can be selected if the team has (i) No girls (ii) At least one girl and one boy? **(05 Marks)**

b. Prove that if 30 dictionaries in a library contain a total of 61337 pages, then atleast one of the dictionaries must have at least 2045 pages. **(05 Marks)**

c. For Fibonacci sequence show that $F_n = \left(\frac{\sqrt{5}+1}{2}\right)^n - \left(\frac{\sqrt{5}-1}{2}\right)^n$ **(06 Marks)**

Module-4

7 a. A card is drawn from a well-shuffled pack of 52 cards. Find the probabilities that the card drawn will be (i) Red (ii) Black queen (iii) King of diamond. **(06 Marks)**

b. For any two events A and B prove that $P(A \cup B) = P(A) \cup P(B) - P(A \cap B)$. **(05 Marks)**

c. Find the probability of two people A and B contradicting each other when they narrate same story given that A speaks 60% true and B speaks 20% false. **(05 Marks)**

OR

8 a. Define event and sample space. State the axioms of probability. **(05 Marks)**

b. Suppose A and B are events with $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.2$. Find the probability that,
(i) A does not occur (ii) B does not occur
(iii) A or B occurs (iv) Neither A nor B occurs **(06 Marks)**

c. A lot contains 12 items of which 4 are defective. Three items are drawn at random from the lot one after the other. Find the probability P that all 3 are non-defective. **(05 Marks)**

Module-5

9 a. Define the following with an example each,
(i) Connected graphs. (ii) Complete graphs (iii) Isomorphic graphs. **(06 Marks)**

b. Show that following graphs are isomorphic. **(05 Marks)**

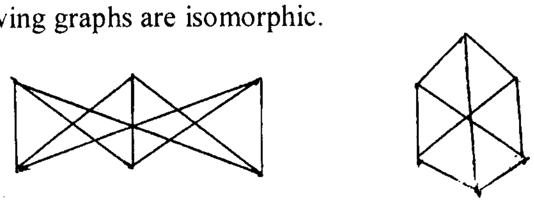

Fig. Q9 (b)

c. Find the chromatic number for each of the cycle of length 4 as shown in the Fig. Q9 (c) Find the chromatic number. **(05 Marks)**

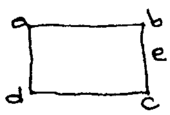

Fig. Q9 (c)

OR

10 a. Define each with an example:
(i) Regular graphs.
(ii) Complement of a graph.
(iii) Bipartite graphs. **(06 Marks)**

b. Prove that a graph G has Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 2n + 6)$ where m is the number o edges, n is the number of vertices and G has no loops or multiple edges $n > 2$. **(05 Marks)**

c. Show that $K_{3,3}$ is non planar. **(05 Marks)**

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